DISTRIBUTED CONSENSUS IN WIRELESS SENSOR NETWORKS WITH QUANTIZED INFORMATION EXCHANGE

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ABSTRACT

In this paper we analyze the impact of quantization on the performance of a discrete-time distributed algorithm aimed at computing the average of an initial set of values in a wireless sensor network. We modify a well-known consensus model and propose a simple scheme where the transmitted data is quantized due to bandwith and/or power constraints. Conversely to existing models that include quantization noise, a closed-form expression for the residual mean square error of the state can be derived for the proposed model. This expression depends on general network parameters and provides therefore an a priori quantitative measure of the effects of quantization on the consensus.

1. INTRODUCTION

Wireless sensor networks (WSN) are typically deployed to sense a physical phenomenon and estimate parameters or detect events. Large deployments are usually built using low cost nodes of reduced processing capabilities and with constraints on bandwidth and power consumption. Although these nodes are designed to perform simple tasks, they can obtain results with the accuracy of the whole network only interacting with neighboring nodes. Consensus algorithms aimed at reaching a common value through local exchange of information are well suited for such deployments (see [1] and references therein). We focus our attention on the well-known scheme by Olfati-Saber and Murray in [2] for the estimation of a parameter, where we assume that the sensors make a measurement and initialize their own state with the sensed value. Using a linear combination of its own previous value and the information received from its neighbors, the state is updated and then broadcasted iteratively. Under given conditions, the system reaches a consensus equal to the average of the initial measurements. Although in a digital implementation of this algorithm the operations are carried out with floating-point precision, it is more realistic to assume that the transmitted information is quantized before it is broadcasted to reduce bandwidth and/or power consumption.

Some contributions found in literature analyze the effects of quantization noise on the consensus. For instance, Xiao *et al* in [3] study the convergence of the model in [2] when the received values are assumed corrupted with an additive noise, and show that the variance of the state vector with respect to the average of the initial values diverges with time. Using the results from [3], Yildiz and Scaglione in [4] exploit time correlation of the data to decrease the variance of the quantization noise when a differential encoder is used. Schizas *et al* propose in [5] distributed MLE and BLUE estimators for the estimation of deterministic signals in *ad hoc* WSNs, where the estimators are formulated as the solution of convex minimization subproblems. Kashyap *et al* introduce in [6] the concept of quantized consensus and propose an algorithm to reach a consensus in that sense. Aldosari and Moura in [7] present a model of consensus based on the work of [2], where each node updates its state using its own value with floating-point precision and the values of its neighbors with quantized precision. Aysal *et al* in [8] present an algorithm based on [2], where the nodes use a probabilistic quantizer. The authors show that the system reaches a consensus whose expected value is equal to the average of the initial values.

In this paper, we consider quantized information exchange among the nodes and propose a discrete-time algorithm based also on the model of [2]. The novelty of the proposed scheme relies on the fact that a closed-form expression for the residual mean square error (MSE) of the state vector is derived, allowing for offline computing. The performance of the algorithm is compared with existing models using computer simulations. The paper is organized as follows. In Section 2 we give some basic concepts of graph theory. Section 3 presents the problem statement and describes the consensus algorithm proposed. The theoretical analysis of the model is included in Section 4 for the case of uncorrelated quantization noise and correlated quantization noise. Section 5 presents the simulation results and Section 6 concludes the paper.

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2. PRELIMINARIES: GRAPH THEORY

The communication flow among the nodes of a network can be described by an undirected graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ where $\mathscr{V} = \{1, \cdots, N\}$ is the set of vertices (nodes) and $\mathscr{E}$ is the set of edges (links) [9]. The edge e_{ij} represents a bidirectional communication link between a pair of distinct nodes i and j. The set of neighbors of node i is defined as $\mathscr{N}_i = \{j \in \mathscr{V} : e_{ij} \in \mathscr{E}\}$ for all $i, j = \{1, \cdots, N\}$, and represents the set of indexes of the nodes sending information to node i. A path in a graph $\mathscr{G}$ is a sequence of alternating nodes and edges that starts and ends at different nodes, such that each edge is incident to its predecessor and successor node. The graph is connected if there exists a path between any two nodes. The connectivity (or topology) of the graph is described by the $N \times N$ Laplacian matrix $\mathbf{L}$, with entries

$$[\mathbf{L}]_{ij} = \begin{cases} d_i^{(\mathrm{out})} & i = j \\ -1 & i \neq j \ \&\ e_{ij} \in \mathscr{E} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $d_i^{(\mathrm{out})}$ stands for the outdegree of node i, and corresponds in our case to $\mathscr{N}_i$. We denote the eigenvalues of $\mathbf{L}$ as $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$, where by definition of $\mathbf{L}$, $\lambda_1 = 0$. In addition, if the graph is connected, from spectral graph theory results we have that the algebraic multiplicity of λ_1 is equal to one and $\mathbf{L}$ is an irreducible matrix [10].

3. PROBLEM STATEMENT

We consider a WSN composed of N nodes, where each node implements a discrete dynamical system whose state evolves according to a difference equation. The state of node i at time instant $k \geq 0$ is a single real number denoted $x_i(k)$, initialized at $x_i(0)$ with the value of the measurement. As proposed by Olfati-Saber and Murray in [2], the state equation for the whole system can be expressed in matrix form as follows

$$\mathbf{x}(k) = \mathbf{W}\mathbf{x}(k-1), \quad \forall k \geq 1 \tag{2}$$

where

$$\mathbf{W} = \mathbf{I} - \epsilon \mathbf{L} \tag{3}$$

is a real symmetric $N \times N$ matrix called the *weight* matrix, $\mathbf{L}$ is the Laplacian of the network defined in (1) and ϵ is a positive constant belonging to the interval $(0, \frac{2}{N-1}]$ to satisfy convergence conditions. It has been shown (e.g. in [11]) that if the matrix $\mathbf{W}$ is symmetric and irreducible, as $k \to \infty$ the vector $\mathbf{x}(k)$ in (2) tends to $\frac{1}{N}\mathbf{1}\mathbf{1}^T\mathbf{x}(0)$, where $\mathbf{1}$ is an all-ones vector of length N and $\mathbf{x}(0) \in \mathbb{R}^{N\times 1}$ is the vector of initial values. We model the initial set of values as real valued Gaussian random variables with variance σ_0^2 and uncorrelated among the nodes, such that $\mathbb{E}[\mathbf{x}(0)] = \mathbf{x}_0$ and $\mathbb{E}\left[\mathbf{x}(0)\mathbf{x}^T(0)\right] = \sigma_0^2\mathbf{I} + \mathbf{x}_0\mathbf{x}_0^T$, where $\mathbb{E}[.]$ is the expected value operator.

We propose a model based on (2) in which each node has access to its own data in floating-point precision, but the state values received from the neighbors are quantized. The state at node i is updated using the quantized state values received from its neighbors, while (conversely to model [7]) its own state value is used both in floating-point precision and quantized as follows

$$x_i(k) = x_i(k-1) + \sum_{j=1, j\neq i}^{N} [\mathbf{W}]_{ij} \left(x_j^q(k-1) - x_i^q(k-1)\right) \tag{4}$$

where $\mathbf{W} = \mathbf{I} - \epsilon\mathbf{L}$ as before, $x_i^q(l) = \psi_q[x_i(l)]$ stands for the quantized value of the state $x_i(l)$, and $\psi_q[\cdot]$ is the function implemented by the quantizer. The state equation in (4) for the whole system can be expressed in matrix form as

$$\mathbf{x}(k) = \mathbf{x}(k-1) - \epsilon\mathbf{L}\mathbf{x}^q(k-1) \tag{5}$$

where $\mathbf{x}^q(l)$ is the quantized state vector at time l. The objective of this paper is to study the impact of the quantization error on the iterative algorithm in (5), assuming that the underlying graph of the network is undirected and connected so that the matrices $\mathbf{L}$ and $\mathbf{W}$ are both symmetric and irreducible. A theoretical approach to evaluate the algorithm in (5) from an analytical point of view consists in modeling the quantization error as an additive noise. For the sake of simplicity, we consider a uniform quantizer with b bits and dynamic range of $[-X_{max}, X_{max}]$. The quantization error can be then modeled as an additive noise uniformly distributed within the interval $(-\frac{\Delta}{2}, \frac{\Delta}{2}]$, with variance $\sigma_q^2 = \frac{\Delta^2}{12}$, where $\Delta = \frac{X_{max}}{2^{b-1}}$ is the quantization step. In that case, the quantized state vector $\mathbf{x}^q(k-1)$ in (5) is equal to

$$\mathbf{x}^q(k-1) = \mathbf{x}(k-1) + \mathbf{e}_q(k-1) \tag{6}$$

where the error vectors $\mathbf{e}_q(l)$ $\forall l > 0$ are assumed zero-mean and independent of the initial values $\mathbf{x}(0)$. Note that this approach is a linearization of the error which gives more tractable mathematical operations. Substituting (6) in (5) we obtain

$$\mathbf{x}(k) = \mathbf{W}\mathbf{x}(k-1) - \epsilon\mathbf{L}\mathbf{e}_q(k-1) \tag{7}$$

Due to the randomness of both the quantization noise and the initial set of measurements, the convergence of $\mathbf{x}(k)$ in (7) must be studied in probabilistic terms. Therefore, we analyze the convergence of the state vector in the mean square sense to a vector $\mathbf{x}_{ave} = \alpha\mathbf{1}$, where $\alpha = \frac{1}{N}\mathbf{1}^T\mathbf{x}_0$. Specifically, we analyze the limit of the MSE of the state vector as $k \to \infty$, i.e.

$$\lim_{k\to\infty} \mathsf{MSE}(\mathbf{x}(k)) = \lim_{k\to\infty} \mathbb{E}\left[\left\|\mathbf{x}(k) - \mathbf{x}_{ave}\right\|_2^2\right]. \tag{8}$$

In the following section, we derive closed-form expressions for the limit in (8) under two different assumptions. First, we assume that the quantization noise is uncorrelated in time, which seems to be a reasonable assumption at the beginning of the iterative algorithm. Afterwards, we consider that the system reaches a stable value so that the quantization noise is correlated in time.

4. PERFORMANCE ANALYSIS

A. UNCORRELATED QUANTIZATION NOISE

Proposition 1. *Consider the system in (7) with $\mathbf{W} = \mathbf{I} - \epsilon\mathbf{L}$ as defined in (3) symmetric and irreducible, $\mathbf{x}(0) \sim \mathcal{N}(\mathbf{x}_0, \sigma_0^2\mathbf{I})$ and $\{\mathbf{e}_q(k); \forall k > 0\}$ a set of i.i.d. zero mean uncorrelated random vectors independent of $\mathbf{x}(0)$, with covariance matrix $\mathbb{E}[\mathbf{e}_q(l)\mathbf{e}_q^T(m)] = \sigma_q^2\delta_{lm}\mathbf{I}$, where δ_{lm} is the Kronecker delta. Under these assumptions, the limit of the MSE of the state vector is*

$$\lim_{k\to\infty} \mathsf{MSE}(\mathbf{x}(k)) = \sigma_0^2 + \sigma_q^2 \sum_{i=2}^{N} \frac{1-\mu_i}{1+\mu_i}. \tag{9}$$

where $\mu_i = 1 - \epsilon\lambda_i$ and λ_i is the i^{th} eigenvalue of $\mathbf{L}$, for all i.

Proof. To compute the limit in (9), we consider the following expression

$$\begin{aligned} \mathsf{MSE}(\mathbf{x}(k)) &= \mathbb{E}\left[\left\|\mathbf{x}(k) - \mathbf{x}_{ave}\right\|_2^2\right] \\ &= \mathrm{var}(\mathbf{x}(k)) + \mathrm{bias}^2(\mathbf{x}(k)) \end{aligned} \tag{10}$$

where the squared bias is

$$\mathrm{bias}^2(\mathbf{x}(k)) = \left\|\mathbb{E}[\mathbf{x}(k)] - \mathbf{x}_{ave}\right\|_2^2 \tag{11}$$

and the variance is

$$\mathrm{var}(\mathbf{x}(k)) = \mathbb{E}\left[\left\|\mathbf{x}(k) - \mathbb{E}[\mathbf{x}(k)]\right\|_2^2\right]. \tag{12}$$

The dynamical system in (7) can be rewritten

$$\mathbf{x}(k) = \mathbf{W}^k\mathbf{x}(0) - \epsilon\sum_{l=1}^{k}\mathbf{W}^{l-1}\mathbf{L}\mathbf{e}_q(k-l) \tag{13}$$

Since $\mathbf{e}_q(k)$ is zero mean $\forall k > 0$, the expected value of $\mathbf{x}(k)$ using (13) is equal to

$$\mathbb{E}[\mathbf{x}(k)] = \mathbf{W}^k\mathbf{x}_0 \tag{14}$$

Then, substituting (14) in (11) and computing the limit we have

$$\lim_{k\to\infty} \mathrm{bias}^2(\mathbf{x}(k)) = \lim_{k\to\infty} \left\|\mathbf{W}^k\mathbf{x}_0 - \mathbf{x}_{ave}\right\|_2^2 = 0$$

where the second equality holds because $\mathbf{W}^k$ tends to $\frac{1}{N}\mathbf{1}\mathbf{1}^T$ as $k \to \infty$, since $\mathbf{W}$ is symmetric and irreducible. In order to compute the variance, we substitute (13) and (14) in (12). Considering $\mathbb{E}[\mathbf{e}_q(l)\mathbf{e}_q^T(m)] = \sigma_q^2\delta_{lm}\mathbf{I}$, after some basic matrix manipulations we obtain

$$\lim_{k\to\infty} \mathrm{var}(\mathbf{x}(k)) = \sigma_0^2 + \sigma_q^2 \lim_{k\to\infty} \mathrm{tr}\Big(\mathbf{W}^{-2}\sum_{l=1}^{k}\mathbf{W}^{2l}\epsilon^2\mathbf{L}^2\Big) \tag{15}$$

where $\mathrm{tr}(.)$ stands for the trace function. To compute the trace in (15) we consider the diagonalization of $\mathbf{L} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$ and $\mathbf{W} = \mathbf{V}\mathbf{M}\mathbf{V}^H$, where $\mathbf{\Lambda}$ and $\mathbf{M}$ are diagonal matrices with entries equal to the eigenvalues of $\mathbf{L}$ and $\mathbf{W}$, respectively, and $\mathbf{V}$ is an Hermitian matrix that diagonalizes both $\mathbf{L}$ and $\mathbf{W}$. Since $\mathbf{W} = \mathbf{I} - \epsilon\mathbf{L}$, the eigenvalues of $\mathbf{W}$ are equal to $\mu_i = 1 - \epsilon\lambda_i$ for all i. The trace in (15) is therefore equal to

$$\mathrm{tr}\Big(\mathbf{W}^{-2}\sum_{l=1}^{k}\mathbf{W}^{2l}\epsilon^2\mathbf{L}^2\Big) = \mathrm{tr}\Big(\mathbf{M}^{-2}\mathbf{M}_k\epsilon^2\mathbf{\Lambda}^2\Big) \tag{16}$$

where we have used the fact that $\mathbf{V}$ is Hermitian and $\mathbf{M}_k$ is a diagonal matrix with entries

$$[\mathbf{M}_k]_{ii} = \begin{cases} k & i = 1 \\ \frac{\mu_i^2 - \mu_i^{2k+2}}{1-\mu_i^2} & i = 2, .., N. \end{cases}$$

Substituting for $\mathbf{M}_k$ in (16) we have

$$\begin{aligned} \mathrm{tr}\Big(\mathbf{W}^{-2}\sum_{l=1}^{k}\mathbf{W}^{2l}\epsilon^2\mathbf{L}^2\Big) &= \sum_{i=1}^{N}\mu_i^{-2}[\mathbf{M}_k]_{ii}\epsilon^2\lambda_i^2 \\ &= \sum_{i=2}^{N}\frac{1-\mu_i}{1+\mu_i}\big(1-\mu_i^{2k}\big) \end{aligned} \tag{17}$$

where in the second equality we have used that $\lambda_1 = 0$. Since $\epsilon \in (0, \frac{2}{N-1}]$, we have $0 \leq \mu_i < 1$. Substituting (17) in (15) and replacing for the variance in (10), we have

$$\lim_{k\to\infty} \mathsf{MSE}(\mathbf{x}(k)) = \sigma_0^2 + \sigma_q^2 \sum_{i=2}^{N} \frac{1-\mu_i}{1+\mu_i}$$

which completes the proof. □

Corollary 1. *The limit of the MSE of the state averaged over N nodes is upper bounded by*

$$\lim_{k\to\infty} \frac{1}{N}\,\mathsf{MSE}(\mathbf{x}(k)) \leq \frac{1}{N}\sigma_0^2 + \frac{N-1}{N}\sigma_q^2 \tag{18}$$

The result in (18) is straightforward, replacing for μ_i and considering $0 \leq \mu_i < 1$ in (9).

Proposition 1 not only shows that the limit of the MSE of $\mathbf{x}(k)$ as $k \to \infty$ exists, but also that this limit can be computed offline, although it would require knowledge of the eigenvalues of $\mathbf{W}$. Additionally, the result from Corollary 1 provides an upper bound for the limit of the MSE of $\mathbf{x}(k)$ that only depends on general parameters of the network, i.e. the number of nodes, the variance of the initial measurements and the variance of the quantization noise.

B. CORRELATED QUANTIZATION NOISE

In this section we analyze the case of correlated quantization noise where we assume that the quantization error is uncorrelated among nodes but is exactly the same from one time instant to the next one, once the state is stabilized.

Proposition 2. *Consider the system in (7) with* $\mathbf{W} = \mathbf{I} - \epsilon\mathbf{L}$ *as defined in (3) symmetric and irreducible,* $\mathbf{x}(0)\sim\mathscr{N}(\mathbf{x}_0, \sigma_0^2\mathbf{I})$ *and* $\{\mathbf{e}_q(k); \forall k > 0\}$ *a set of i.i.d. zero mean correlated random vectors independent of* $\mathbf{x}(0)$*, with covariance matrix* $\mathbb{E}[\mathbf{e}_q(l)\mathbf{e}_q^T(m)] = \sigma_q^2\mathbf{I}$*. Under these assumptions, the limit of the* MSE *of the state vector is*

$$\lim_{k\to\infty} \mathsf{MSE}(\mathbf{x}(k)) = \sigma_0^2 + (N-1)\sigma_q^2. \tag{19}$$

Proof. Following a similar procedure as in Proposition 1, we have that the bias is 0 and the limit of the variance is

$$\lim_{k\to\infty} \mathrm{var}(\mathbf{x}(k)) = \sigma_0^2 + \sigma_q^2 \lim_{k\to\infty} \mathrm{tr}\Big(\sum_{l=1}^{k}\sum_{m=1}^{k} \mathbf{W}^{m-1}\mathbf{W}^{l-1}\epsilon^2\mathbf{L}^2\Big) \tag{20}$$

which is slightly different from (15) because now we have $\mathbb{E}[\mathbf{e}_q(l)\mathbf{e}_q^T(m)]{=}\sigma_q^2\mathbf{I}$. Using the spectral decomposition of $\mathbf{L}$ and $\mathbf{W}$ as in (16), the trace in (20) is equal to

$$\begin{aligned}\mathrm{tr}\Big(\sum_{l=1}^{k}\sum_{m=1}^{k} \mathbf{W}^{m+l-2}\epsilon^2\mathbf{L}^2\Big) &= \mathrm{tr}\Big(\sum_{l=1}^{k}\sum_{m=1}^{k} \mathbf{M}^{m+l-2}\epsilon^2\mathbf{\Lambda}^2\Big)\\ &= \sum_{i=2}^{N}\epsilon^2\lambda_i^2\mu_i^{-2}\left(\frac{\mu_i-\mu_i^{k+1}}{1-\mu_i}\right)^2\end{aligned} \tag{21}$$

As $0 \le \mu_i < 1$ for $i = 2, \dots N$, the term μ_i^{k+1} in the expression above tends to zero as $k \to \infty$. The limit of the trace in (21) is given by

$$\begin{aligned}\lim_{k\to\infty}\sum_{i=2}^{N}\epsilon^2\lambda_i^2\mu_i^{-2}\left(\frac{\mu_i-\mu_i^{k+1}}{1-\mu_i}\right)^2 &= \sum_{i=2}^{N}\frac{\epsilon^2\lambda_i^2}{(1-\mu_i)^2}\\ &= (N-1).\end{aligned}$$

Then, replacing these results in (20) and substituting for the variance in (10), the limit of the MSE of $\mathbf{x}(k)$ is

$$\lim_{k\to\infty} \mathsf{MSE}(\mathbf{x}(k)) = \sigma_0^2 + (N-1)\sigma_q^2$$

which completes the proof. □

Note that assuming a correlated quantization error, the limit of the MSE of $\mathbf{x}(k)$ coincides with the upper bound derived in (18) if we average over N nodes.

5. SIMULATION RESULTS

In this section we use computer simulations to evaluate the performance of the model proposed in (5) implementing the quantizer $\mathbf{x}^q(k) = \psi_q[\mathbf{x}(k)]$. For the sake of comparison, two similar models are additionally simulated (see Table 1). The floating-point precision model in (2), denoted “No quant” in the table, is also included as a benchmark. Model 1 assumes that the updates are carried out in every iteration using only the quantized states. Model 2 corresponds to the model proposed in [7], and assumes that each node updates its state using its own previous value with floating-point precision and the state values of the neighbors in quantized precision ($\mathbf{W}_D$ in Table 1 denotes a diagonal matrix whose entries are equal to the diagonal entries of $\mathbf{W}$). We consider a WSN composed of $N = 20$ nodes randomly deployed in a rectangular area of dimensions 100×100, where the neighbors for each node are all the nodes inside a circle of radius $a = 35$ centered at the former node. The network used in the simulations is connected with $\epsilon = \frac{2}{N-1}$. The initial measurements are modeled as Gaussian random variables with mean $\mathbf{x}_0 = x_0\mathbf{1}$ and variance $\sigma_0^2 = 25$, uncorrelated among nodes. We have observed that the performances of the three models depend on how close the mean value x_0 is to a quantized value. For this reason, x_0 is selected at random within the range $[-15, +15]$ and different in every realization. We implement a uniform symmetric quantizer operating in the range of $[-20, +20]$ with $b = 3$ and $b = 6$ quantization bits. Figures 1 and 2 show the evolution of the MSE in equation (8) in dB as a function of the iterations, averaged over all nodes and 10000 independent realizations for the models in Table 1 with $b = 3$ and $b = 6$ respectively. The simulations show that the MSE of the state converges for all models and, as expected, the performance improves with a higher number of bits. The figures also plot the limits obtained in (9) and (19), along with the theoretical benchmark of σ_0^2/N. As expected, the system with floating-point precision reaches the theoretical bound of σ_0^2/N asymptotically. Note that for the case of $b = 6$, the limits obtained in (9) and (19) approach each other, and they both approach the theoretical benchmark. We observe that the performance of Model 1 is the worst even with a relatively high number of quantization bits. Comparing the performance of Model 2 and Model 3, our model attains a smaller MSE in both cases. From simulations not included here due to lack of space, we have observed that if the number of quantization bits is greater than 6, all models in Table 1 behave quite

No quant	$\mathbf{x}(k) = \mathbf{W}\mathbf{x}(k-1)$
Model 1	$\mathbf{x}(k) = \mathbf{W}\mathbf{x}^q(k-1)$
Model 2	$\mathbf{x}(k) = \mathbf{W}_D\mathbf{x}(k-1) + (\mathbf{W}-\mathbf{W}_D)\,\mathbf{x}^q(k-1),$
Model 3	$\mathbf{x}(k) = \mathbf{x}(k-1) - \epsilon\mathbf{L}\mathbf{x}^q(k-1)$

Table 1. Simulated consensus models with $\mathbf{x}^q(k) = \psi_q[\mathbf{x}(k)]$.

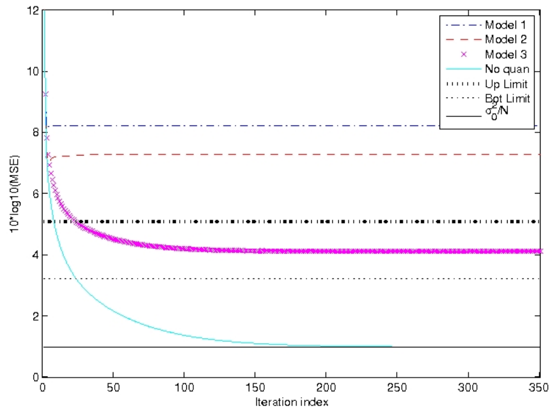

Fig. 1. MSE of the state in dB averaged over $N = 20$ nodes with $b = 3$ quantization bits.

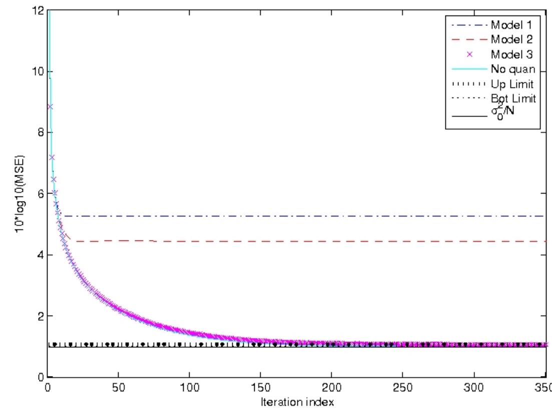

Fig. 2. MSE of the state in dB averaged over $N = 20$ nodes with $b = 6$ quantization bits.

similar. Moreover, we have simulated networks with up to 50 nodes and, in all cases, the MSE of the state remains between the limits derived previously.

6. CONCLUSION

A model to achieve the average consensus in a WSN where the information exchanged among nodes is quantized has been presented. The simulations show that, when a uniform quantizer is used, the proposed model outperforms similar existing consensus models that also include quantization. Conversely to these models, the limit of the MSE of the state for the proposed model exists, and under certain assumptions of the quantization error, a closed-form expression for this limit is derived. An upper bound for this expression that depends only on general network parameters is also found. This upper bound might be useful in the design of the quantizer implemented by the nodes.

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