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Control of bistability in a directly modulated semiconductor laser using delayed optoelectronic feedback

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Abstract

We show numerically that direct delayed optoelectronic feedback can suppress hysteresis and bistability in a directly modulated semiconductor laser. The simulation of a laser with feedback is performed for a considerable range of feedback strengths and delays and the corresponding values for the areas of the hysteresis loops are calculated. It is shown that the hysteresis loop completely vanishes for certain combinations of these parameters. The regimes for the disappearance of bistability are classified globally. Different dynamical states of the laser are characterized using bifurcation diagrams and time series plots.

Keywords: Directly modulated feedback; Bistability; Delayed optoelectronic feedback

1. Introduction

Semiconductor lasers are widely used as the coherent light sources for technological applications such as high capacity optical transmission and ultra fast optical processing. They are preferable to any other types of lasers in the field of optical communications because of their compactness, low cost and convenience of operation. Another advantage of semiconductor lasers is the opportunity for direct modulation, i.e., they can be modulated by varying their injection current. The modulation is usually performed in the GHz frequency domain. It is well known that semiconductor lasers show highly complex phenomena such as sub-harmonic generation, chaos and multistability under strong modulation in this frequency range [1]. A clear understanding of such phenomena would be helpful in the design of optical communication systems using directly modulated laser diodes. Further, it may help us in employing suitable control schemes for such lasers.

The modulation response of lasers has been investigated by several groups in the last two decades [2–9]. For small signal modulation, semiconductor lasers show regular behavior even if the frequency of modulation is of the order of a few GHz (of the order of their relaxation oscillations) [10]. However, if the strength of the modulating signal is increased, the laser follows a period-doubling route to chaos [2–8]. Another important dynamical behavior shown by the laser under direct modulation is multistability [8]. The coexistence of two or more stable states for a dynamical system is commonly referred to as multistability. In this case, the asymptotic state of the system is critically determined by its initial conditions. The attractors corresponding to the different dynamical states (such as chaotic and periodic states) can coexist. This phenomenon is referred to as generalized multistability [11] and has been reported in various nonlinear systems such as lasers [11–14], Duffing oscillators [15], electronic circuits [16] and biological systems [17,18]. In modulated semiconductor lasers, multistability is usually associated with the hysteresis effect [19]. The coexistence of two distinct states is very common and is referred to as bistability of the laser [1].

Since the laser is expected to operate in a regular and unique dynamical state, chaos and bistability should be eliminated from the laser. Recently, we have shown that a direct delayed optoelectronic feedback can suppress the sub-harmonic generation and chaos in directly modulated laser diodes [20]. Bidirectional coupling of lasers is also found to be efficient in

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suppressing chaos in these lasers [21,22]. Further, if two states are stable, noise induced switching is possible between these states [23]. For these reasons, investigations of the control of bistability in lasers also deserve considerable attention.

Many attempts have been reported for suppressing multistability in various nonlinear dynamical systems [24–32]. A common technique for controlling generalized multistability is the annihilation of unwanted stable states by applying small harmonic modulation with a properly chosen frequency and amplitude to a system parameter. The control of multistability using periodic perturbation has been investigated in different theoretical models such as the Hénon map [24–28], laser models [24,28], coupled Duffing oscillators [29] and a delayed logistic map [28,30]. This method has been demonstrated experimentally using a cavity loss modulated CO_2 laser [24] and an erbium doped fiber laser [31]. Recently, Pisarchik and Kuntsevich have shown that the method of periodic perturbations can also be applied for controlling multistability in a directly modulated semiconductor laser [32]. Another way of annihilating coexisting attractors is to add noise. The stable fixed points of the Lorenz system have been found to be annihilated when noise is added to the system [33].

In this paper, we propose a method for controlling bistability in directly modulated laser diodes without applying perturbation from an additional modulating source. We show numerically that a direct delayed optolectronic feedback with suitable delay and feedback strength can effectively suppress the hysteresis and bistability in directly modulated semiconductor lasers. One of our interesting results is that the delay, as well as the feedback strength, plays a very important role in the suppression of bistability.

The idea of the delay feedback control of chaos was originally proposed by Pyragas [34]. Currently this method is known as Time Delay Auto Synchronization (TDAS) and has shown to be successful in controlling chaos in various chaotic systems such as electronic circuits [35–37], glow discharge [38], magneto-elastic ribbon [39], periodically driven yttrium iron garnet film [40] and CO_2 lasers [41]. Ciofini et al. have used a suitable implementation of the TDAS method by means of a tunable washout filter for controlling high-dimensional chaos in such lasers [42]. The basic criterion of the TDAS algorithm is to stabilize a particular unstable periodic orbit (UPO), which is embedded in the chaotic attractor by applying a time continuous perturbation to an accessible variable of the system. This perturbation must be proportional to the difference between the present value of a system variable and the value of the same variable corresponding to an earlier state of the system. The delay must be equal to the period of the specific UPO which is to be stabilized.

The concept of a direct delayed feedback [20] is entirely different from that of a self-adjusting feedback, which is used in a TDAS algorithm for stabilizing the unstable periodic orbits. In direct delayed feedback, a perturbation proportional to the delayed output signal of the system is added to the input of the system. For a modulated semiconductor laser, the output is the optical signal. It should be converted to an electronic signal using a photo diode, and this signal can be added to the injection

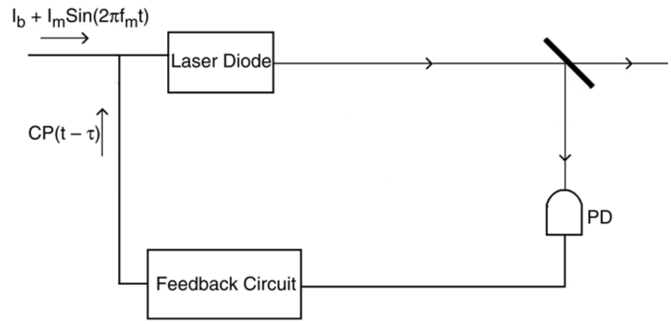

Fig. 1. Schematic diagram of laser with delayed optoelectronic feedback.

current with an appropriate delay. A simple way of producing the delay is to allow the signal to travel a certain distance before reaching the photo diode. There is no general rule for selecting the delay. We can determine the appropriate values of delay and feedback strength by simulating the system for a range of values of these parameters. Unlike in the TDAS method, the perturbation applied to the system need not vanish when control is achieved. Due to the same reason, the periodic orbit of the controlled system will differ from the original UPO present in the phase space of the uncontrolled system. This method will work even in the absence of a UPO of any specific period in the attractor since the control does not depend on the stabilization of any existing UPO. New periodic states replace the chaotic state when the feedback is applied. An important advantage of the delayed optoelectronic feedback scheme is that, besides suppressing chaos and period doubling in a laser, it eliminates the double peak structure of the original periodic orbits in the laser diode.

In contrast to the situation described in the control of chaos, our task is not to stabilize any unstable periodic orbit associated with the laser. Instead, we have investigated the possibility of having a single and unique attractor generated as a result of the delay feedback. Our simulations show that the newly generated attractor replaces the coexisting attractor if the delay and feedback strength are chosen properly. In the method prescribed by Pisarchik and Kuntsevich, the unwanted stable orbit is selectively destroyed using a resonant periodic perturbation (perturbation with a frequency which is close to the characteristic frequency of the specific periodic orbit to be annihilated, i.e., the imaginary part of complex eigenvalues of the stable periodic cycle). The method presented in our work does not require a second modulation, and the perturbation applied here is simply a delayed feedback of the laser.

2. Model of the laser with feedback

A schematic diagram of the laser diode with the control set up is given in Fig. 1. The delayed optoelectronic feedback prescribed in this diagram has been studied widely for different applications such as the generation of ultra short optical pulses [43,44] and chaotic signals [45]. In this method, the light signal from the laser diode (LD) is converted into an electronic signal using a high-bandwidth photodiode (PD). The resultant signal is then amplified to the required gain and is fed back with the input injection current of the laser. A delay is also incorporated with the feedback. The required delay can be provided by the external transit of the light signal by a predetermined distance.

The rate equations of a single mode semiconductor laser are given by [10]

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{I}{qV} - \frac{n}{\tau_e} - A(n - n_0)p, \tag{1}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \Gamma A(n - n_0)(1 - \varepsilon_{\mathrm{NL}} p)p - \frac{p}{\tau_p} + \Gamma\beta\frac{n}{\tau_e}, \tag{2}$$

where n and p are the carrier and photon densities, τ_e and τ_p are the carrier and photon life times, I is the injection current, q is the electronic charge, V is the volume of the active region, n_0 is the carrier density for transparency (the electron density above which the lasing gain becomes positive), A is the gain constant, β is the spontaneous emission factor, Γ is the confinement factor, and $\varepsilon_{\mathrm{NL}}$ is the constant governing the nonlinear gain reduction occurring with an increase in p due to the nonlinear effects such as spectral hole burning.

For the convenience of numerical study, the normalized carrier density N and the normalized photon density P can be used. These are defined by [9]

$$N = \frac{n}{n_{\mathrm{th}}}, \qquad P = \frac{p}{p_0}, \tag{3}$$

where $n_{\mathrm{th}} = n_0 + (\Gamma A\tau_p)^{-1}$, the threshold carrier density, and $p_0 = \Gamma(\tau_p/\tau_e)n_{\mathrm{th}}$.

The rate equations then become

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{1}{\tau_e}\left(\frac{I}{I_{\mathrm{th}}} - N - \frac{N-\delta}{1-\delta}P\right), \tag{4}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{\tau_p}\left(\frac{N-\delta}{1-\delta}(1-\varepsilon P)P - P + \beta N\right), \tag{5}$$

where $\delta = n_0/n_{\mathrm{th}}$ and $\varepsilon = \varepsilon_{\mathrm{NL}} p_0$ are two dimensionless parameters and $I_{\mathrm{th}} = qVn_{\mathrm{th}}/\tau_e$ is the threshold current of the laser.

We assume that a sinusoidal modulation is applied to the laser in the form

$$I = I_b + I_m \sin(2\pi f_m t), \tag{6}$$

where I_m and f_m is the amplitude and frequency of modulation, respectively. To represent the delay feedback applied to the laser, we must add a perturbation term $CP(t-\tau)$ to the injection current given by Eq. (6). This term is proportional to the photon density of the laser corresponding to the past state of the laser, where C is the feedback strength and τ is the delay. Then the injection current can be expressed as

$$I = I_b + I_m \sin(2\pi f_m t) + CP(t-\tau). \tag{7}$$

We have numerically solved the dynamical equations (Eqs. (4), (5) and (7)) of the laser with delay feedback using a fourth order Runge–Kutta algorithm. The parameters of InGaAsP semiconductor lasers used for the simulation are given in Table 1. InGAsP laser diodes are widely used in optical communications, since they emit light in the wavelength region where the optical fibers yield minimum loss to the transmitted light [10].

Table 1
Parameter values of InGaAsP laser diodes used for simulation

τ_e	3 ns
τ_p	6 ps
δ	0.692
ε	10^{-4}
β	5×10^{-5}
I_b	26 mA
f_m	0.8 GHz

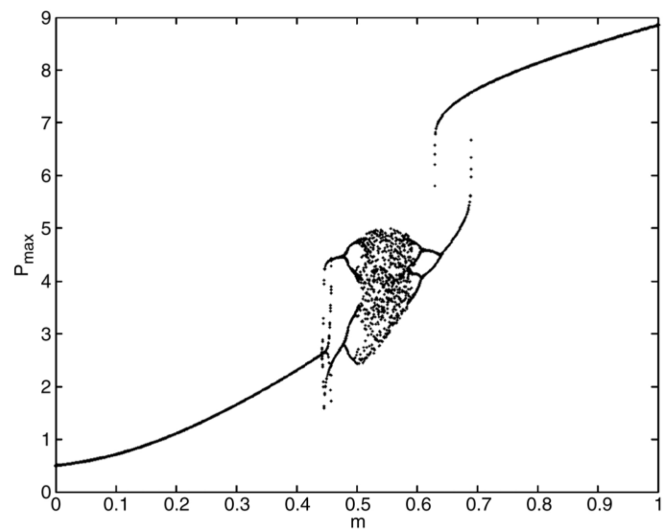

Fig. 2. Bifurcation diagram (maxima of normalized photon density versus modulation strength) of the modulated laser without feedback obtained by continuous-time simulation. The modulation depth is varied from 0 to 0.8 and vice versa.

3. Results and discussions

The dynamics of the directly modulated laser have already been studied in detail [2–9]. However, we start our discussion by presenting the numerical results regarding the complex phenomena in a directly modulated laser diode, i.e., the period-doubling route to chaos and the coexistence of multiple attractors. It would be helpful to understand the necessity of controlling these effects. The onset of chaos and hysteresis in a semiconductor laser modulated by a sinusoidal signal of frequency $f_m = 0.8$ GHz is demonstrated using the bifurcation diagram (maxima of the normalized photon density versus modulation strength $m = I_m/I_{\mathrm{th}}$) given in Fig. 2. It is assumed that the feedback is not applied to the laser and hence C is taken to be 0 in the simulation. The advantage of taking maxima is that it shows the variation of peak power with an increase in the amplitude of modulation, in addition to showing the variation of the periodicity of the pulses. We have taken only one maximum in each cycle instead of taking all maxima. This is because of the double peaked pulses, and taking all maxima will show a number of attractor points that is greater than the periodicity of a particular cycle. In the simulation of the laser, it is assumed that the modulation begins with a minimum value of amplitude,

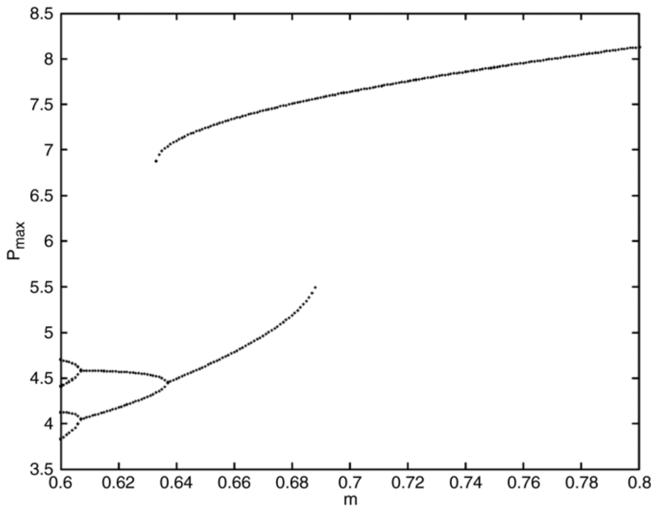

Fig. 3. Bifurcation diagram (maxima of normalized photon density versus modulation strength) of the laser without feedback plotted using the continuous-time method. A detailed view of the hysteresis in the periodic regime is clearly shown.

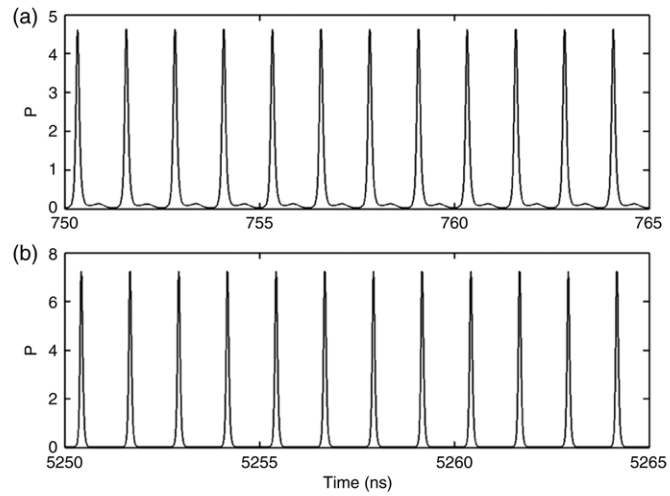

Fig. 4. Time series plots of the normalized photon density of the laser without feedback for a modulation strength of 0.65 correspond to the (a) lower and (b) upper branch of the hysteresis loop.

and that the amplitude is gradually increased up to a maximum value while the laser is operating. It is supposed to be decreased gradually to the minimum in the same manner. This method of constructing a bifurcation diagram is called the continuous time approach, and the bifurcation diagrams constructed by this method will show the hysteresis effect correctly [46]. Initial conditions are given only in the beginning of the simulation, unlike the usual method called the brute force approach, in which the initial values are given every time when the parameter is changed.

For small values of modulation strength, the laser operates in the period 1 state. Period doubling bifurcations start at $m = 0.449$ and continue up to about $m = 0.55$, where the system is perfectly chaotic. On increasing the strength of modulation beyond this value, reverse period doubling takes place and the laser becomes periodic. For a modulation strength greater than 0.639, the laser operates in the period 1 state. Besides showing the period doubling route to chaos, the bifurcation diagram given in Fig. 2 clearly presents the hysteresis effect. When the modulation depth is increased near the point $I_m = 0.49$ and $I_m = 0.61$, the peak photon densities increase though a particular path. As the modulation strength is decreased again, the peaks follow a different path. Thus, the graph of the peak photon densities forms a closed curve — hysteresis loop — in those regimes. A detailed view of the hysteresis loop formed between $m = 0.6$ and $m = 0.8$ is given in Fig. 3. It shows that, on increasing the modulation depth from 0.6 to 0.8, the variation in the peak photon densities takes place through the lower path in the figure. The modulation depth is assumed to decrease after reaching the value 0.8. The return path deviates from the original path at the point $m = 0.689$, follows another route over the lower path, and forms the hysteresis loop. The loop ends at $m = 0.631$, where the period 2 solution exists. It is clear from Fig. 3 that there will be two stable solutions for the laser diode for any value of modulation depth between 0.631 and 0.689, and two types of optical pulses can be obtained. For example Fig. 4(a) and (b), respectively, show time series plots of two different optical pulses obtained for the same value of modulation strength ($m = 0.65$). Fig. 4(a) corresponds to the lower branch and is obtained while the modulation was increasing from 0.6 to 0.8. The peak power of this pulse is 4.631 units. The pulse corresponding to the upper branch (Fig. 4(b)) is obtained when the modulation is decreased again from 0.8 to 0.6 and its peak power is 7.241 units. Hence, it is clear that the laser shows bistable behavior for these parameters.

We now consider the effect of delayed optoelectronic feedback on the bistable behavior of the laser. We have simulated the modulated semiconductor laser with delay feedback for a range of values of the feedback strength and delay. To study quantitatively the effect of feedback on bistability, we have calculated the area of the hysteresis loop shown by the laser for different combinations of delay and feedback strength. The parameter space constituted by the delay and feedback strength is divided into three regimes, and they are shown in Fig. 5. The unshaded regions give the values of parameter for which the hysteresis has not vanished. For certain combinations of C and τ, the area of the hysteresis loops becomes very small (below 10^{-4} units) and hence negligible. Such combinations are given by the dotted regions in the diagram. The shaded regions correspond to the values of parameters for which the hysteresis loop has completely vanished. It is obvious from Fig. 5 that there are three such regions in parameter space, and these are covered by the regions of partial suppression of hysteresis. One of the shaded portions corresponds to positive feedback. It is clear that, if we are applying a positive feedback, relatively higher values of feedback strength are required to suppress the hysteresis compared to the negative feedback.

The area of the hysteresis loop is plotted as a function of the absolute value of the feedback strength in Fig. 6 (negative feedback is considered here). The delay is taken to be a constant value of 0.4 ns. The global classification diagram given by Fig. 5 suggests that a complete elimination

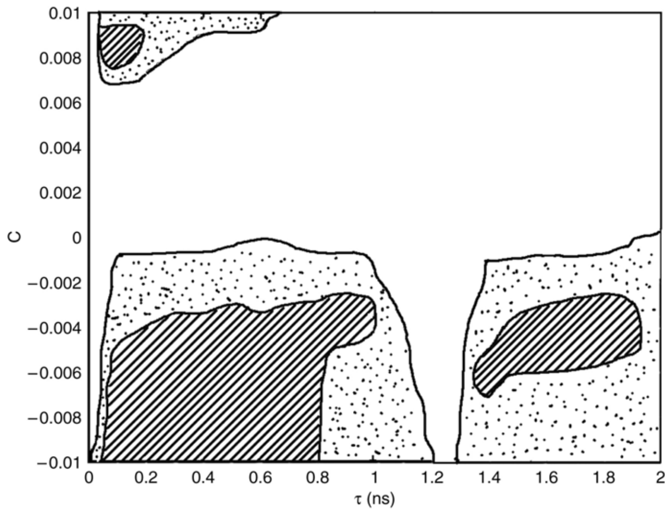

Fig. 5. Regimes of delay and feedback strength, where the bistability has been suppressed. The portions shaded with lines correspond to the complete disappearance of the hysteresis loop. The dotted portions show the regions where the area of the loop becomes less than 10^{-4} units.

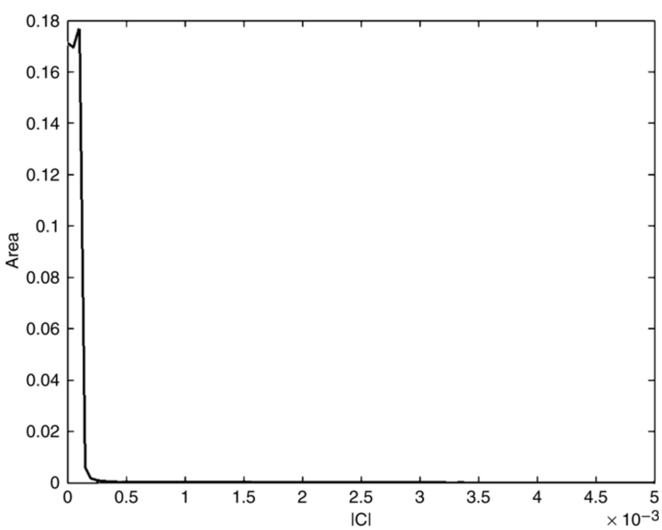

Fig. 6. Variation of the area of hysteresis loop versus the absolute value of feedback strength. Feedback is negative with a delay $\tau = 0.4$ ns. Notice that the area of the loop becomes almost negligible with a feedback of very low values of feedback strength.

of hysteresis is possible for this value of delay if we adjust the feedback strength properly. The area decreases very rapidly as the feedback strength is increased. The area becomes almost negligible for very small values of feedback strength around -6×10^{-3}. It vanishes completely for all values of feedback strength greater than 4×10^{-3} (absolute value).

The vanishing of the hysteresis loop is a consequence of the disappearance of bistability too. The bifurcation diagram (Fig. 7) shows the variation of the maxima of the normalized photon density of the laser with feedback ($\tau = 0.4$ ns and $C = -0.005$). The upper and lower branches of the hysteresis loop exactly coincide with each other. In this case there is only one unique dynamical state for the laser corresponding to any particular value of the amplitude of modulation. The

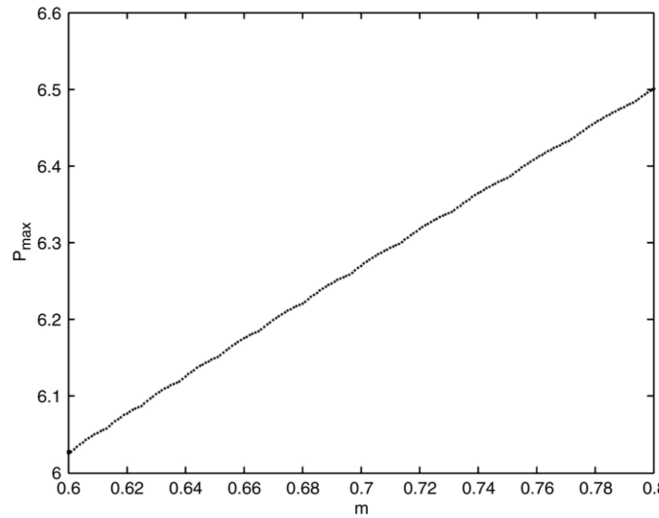

Fig. 7. Variation in the peaks of the normalized photon density of the laser with an increase in modulation strength. A delayed negative feedback is applied to the laser. $\tau = 0.4$ ns and $C = -0.005$. The upper and lower branches of the hysteresis coincide with each other, showing that the bistability has disappeared.

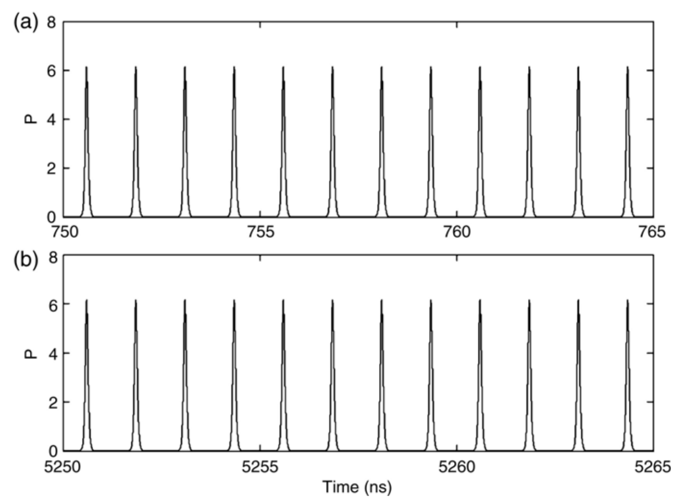

Fig. 8. Time series plots of the normalized photon density of the laser with a delayed negative feedback. $m = 0.65$, $\tau = 0.4$ ns and $C = -0.005$. The pulses obtained while (a) increasing (b) decreasing the modulation strength. The pulses are similar and they correspond to the new dynamical state produced by feedback.

time series plot of the pulses generated during an increase and a decrease in modulation strength for these values of feedback parameters are given in Fig. 8. The modulation strength is 0.65 for both these pulses. Fig. 8(a) and (b) show the pulse obtained while increasing and decreasing the amplitude of modulation, respectively. Peak powers of both of these pulses are the same ($P_{\max} = 6.149$) and it is different from the peak powers of the pulses corresponding to the lower and upper branches of the hysteresis loop of the laser without feedback (Fig. 4(a) and (b)). Hence, it is evident that a new dynamical state has replaced the multiple dynamical states corresponding to a single value of modulation strength.

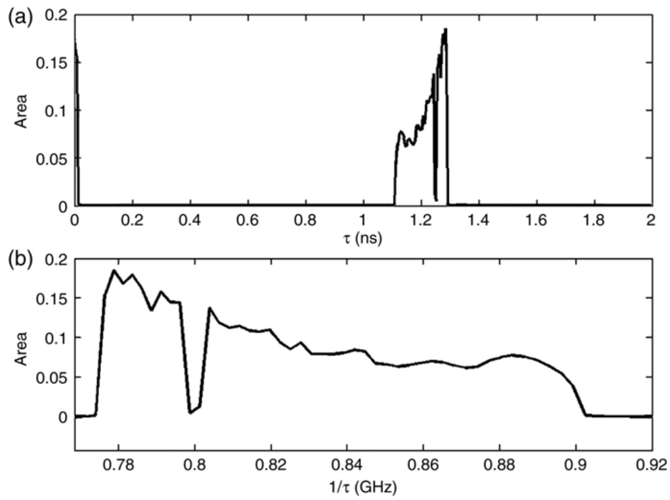

Fig. 9. (a) Variation in the area of the hysteresis loop with an increase in delay. (b) The peak region of the area showing delay-induced bistability. The area of the hysteresis loop is plotted against the inverse of the feedback delay. $C = -0.004$.

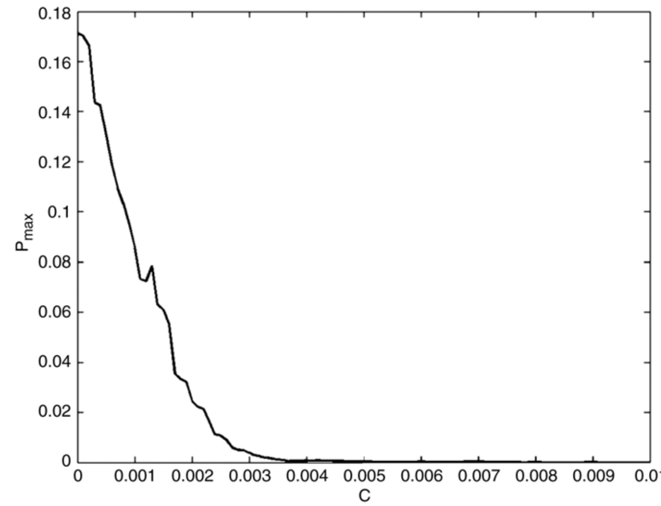

Fig. 10. Variation in the area of the hysteresis loop of the laser with an increase in feedback strength. Feedback is positive here. $\tau = 0.05$ ns.

A very interesting result of our simulation is the dependence of the suppression of bistability on the delay. It can be seen from Fig. 5 that the complete elimination of hysteresis is possible only with a feedback of non-zero delay. Further, there is a regime of feedback strength where there is no possibility of even a partial vanishing of the hysteresis loop. The variation in the area of the hysteresis loop with an increase in delay is given in Fig. 9(a). The feedback strength is kept to be a constant -0.004. This feedback strength is sufficient to eliminate the bistability according to Fig. 5. The area becomes negligible around $\tau = 0.02$ ns. On further increasing the delay, it will completely vanish at $\tau = 0.0235$ ns. Considerably large hysteresis loop is formed for the delays that lie between 1.1 and 1.3 ns. This implies that the feedback induces bistability in this range of delay. It is interesting to examine how this frequency range is related to the frequency of the coexisting periodic orbits. Fig. 9(b) shows a detailed view of the peak observed in Fig. 9(a). Here, the area of the hysteresis loop is plotted against the inverse of the feedback delay. The peak is formed in the range 0.77–0.91 GHz, which is around the frequency of the coexisting period 1 cycles, i.e., the frequency of modulation (0.8 GHz). After increasing the delay further, the area of the loop will vanish again. The above results show that the elimination of bistability depends on delay.

The suppression of chaos is possible with positive feedback too. However, the required feedback strength is relatively high in this case. Fig. 10 gives the variation in the area of the hysteresis loop with an increase in feedback strength. The delay is chosen to be equal to 0.05 ns. Fig. 5 shows that the suppression of bistability using positive feedback is possible for this value of delay. The area becomes negligibly small around the value 0.0056, and complete disappearance takes place at $C = 0.0074$. There will be no hysteresis for the laser if a feedback of strength greater than this value is applied. Fig. 11 shows the hysteresis loops corresponding to different values of feedback strengths ($\tau = 0.05$ ns). For $C = 0.0003$, the area of the hysteresis loop (Fig. 11(a) is equal to 0.1437 units. (The area of the loop in the absence of feedback is found to be equal to 0.1712 units.) The shape of the loop has changed and the area has reduced to 0.0609 units when a feedback of strength 0.0015 is applied (Fig. 11(b)). The size of the loop has again decreased, and the area has become 0.0113 when C is increased to 0.0024 (Fig. 11(c)). Fig. 11(d) shows that the hysteresis has almost disappeared for $C = 0.005$. The area of the loop corresponding to this value of feedback strength is 3.35×10^{-4}. The upper and lower branches of this loop with extremely small area cannot be distinguished in the figure. Fig. 12 shows the complete bifurcation diagram of the laser with a feedback of delay 0.05 ns and strength 0.008 obtained by the continuous time approach. The variation in the peak photon density is smooth and the hysteresis loop has disappeared completely. All types of bifurcations shown in Fig. 2 are absent here. We have already shown that this combination of feedback strength and delay gives a complete elimination of period doubling and chaos from the laser [20]. The results presented here show that the same feedback can efficiently suppress the hysteresis too.

4. Conclusion

The effect of delayed optoelectronic feedback on bistable behavior is demonstrated numerically. We have shown that this type of feedback is a simple and efficient method for controlling bistability. The regimes of feedback parameters corresponding to the complete elimination of hysteresis and bistability are found out by simulating the system for a range of values of feedback parameters. It is shown that the elimination of bistability is possible with feedback of very small strength if a delayed negative feedback is used. In the practical sense, direct delayed optoelectronic feedback is a very easy way of keeping the laser in a unique and regular dynamical state. The mechanism of destabilization of one of the bistable states deserves particular attention and can be investigated in detail in the future. This may reveal more interesting features of the dynamics of lasers with delayed feedback.

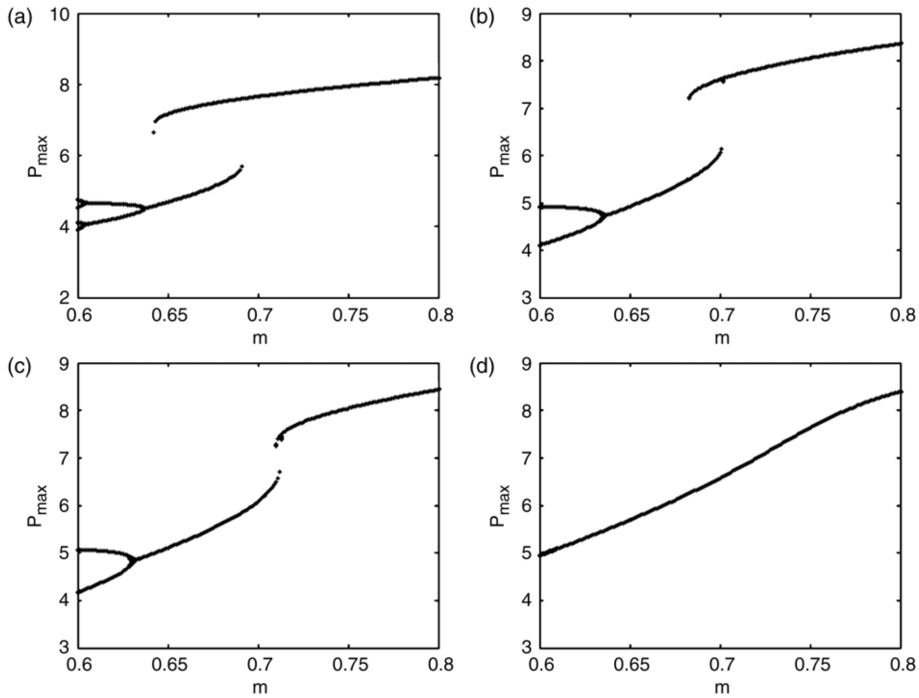

Fig. 11. Variation in the structure of the hysteresis loops with the increase of feedback strength. A positive feedback is considered here. (a) $C = 0.0003$, (b) $C = 0.0015$, (c) $C = 0.0024$, (d) $C = 0.005$.

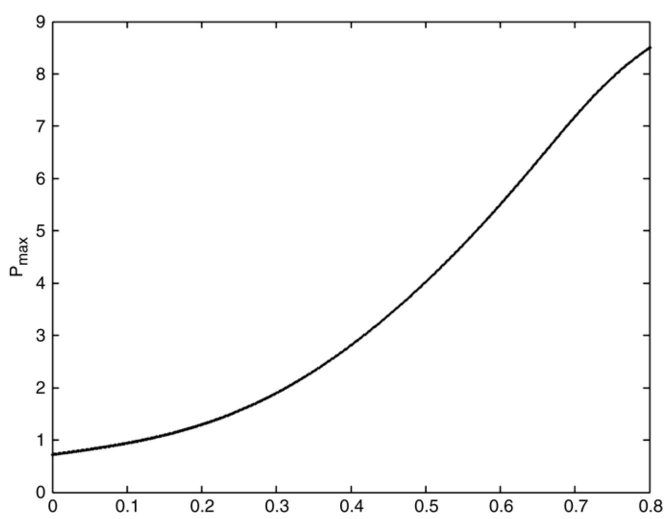

Fig. 12. Bifurcation diagram showing the peaks of the normalized photon density of the laser with an increase in modulation depth. A delayed positive feedback is considered here. $C = 0.008$ and $= 0.05$ ns. It is plotted using the continuous time approach. The coincidence of forward and reverse paths shows that the bistability is completely eliminated.

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