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Joghee Ravichandran
Amrita Vishwa Vidyapeetham, Amrita University, aishwar2@rediffmail.com

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A Note on Determination of Sample Size from the Perspective of Six Sigma Quality

Joghee Ravichandran
Amrita Vishwa Vidyapeetham, Amrita University
Coimbatore, India

In most empirical studies (clinical, network modeling, and survey-based and aeronautical studies, etc.), sample observations are drawn from population to analyze and draw inferences about the population. Such analysis is done with reference to a measurable quality characteristic of a product or process of interest. However, fixing a sample size is an important task that has to be decided by the experimenter. One of the means in deciding an appropriate sample size is the fixation of error limit and the associated confidence level. This implies that the analysis based on the sample used must guarantee the prefixed error and confidence level. Although there are methods to determine the sample size, the most commonly used method requires the known population standard deviation, the preset error and the confidence level. Nevertheless, such methods cannot be used when the population standard deviation is unknown. Because the sample size is to be determined, the experimenter has no clue to obtain an estimate of the unknown population standard deviation. A new approach is proposed to determine sample size using the population standard deviation estimated from the product or process specification from the perspective of Six Sigma quality with a goal of 3.4 defects per million opportunities (DPMO). The aspects of quality improvement through variance reduction are also presented. The method is effectively described for its use and is illustrated with examples.

Keywords: Coefficient of variation, DPMO, error, confidence level, sample size, Six Sigma quality, stopping criteria

Introduction

In most empirical studies, sample observations are often used to analyze and draw inferences about the population. Though a larger sample size results in better conclusions, the choice of sample size is very important for such studies. This is due to the fact that a larger sample size may require too much time, resources, and cost and, at the same time, a smaller sample size may lead to inaccurate inferential

Joghee Ravichandran is a Professor in the Department of Mathematics, Amrita School of Engineering. Email him at: aishwar2@rediffmail.com.

results. Therefore, in practice, before the choice of sample size, the aspects of time, resources, and cost have to be taken into consideration in addition to sufficient statistical power. An experimenter also prefers to fix a sample size without much compromise on the two types of errors. The problem of sample size determination is quite common in the research areas such as clinical trials (Ando et al., 2015), network modeling (Krivitsky & Kolaczyk, 2015), and aeronautical studies (Suárez-Warden, Rodriguez, Hendrichs, García-Lumbreras, & Mendívil, 2015).

In order to know how large a sample size must be fixed, a number of factors may be considered by both statisticians and researchers. Sometimes, it depends on the nature of study of interest. That is, the study may be survey-based to find out the proportion of something, or may be to estimate the population mean, standard deviation, correlation coefficient, regression coefficients, etc. So, given the nature of a study, "how to conclude if the sample size used is enough and is the right representation of the population?" is the most commonly raised question.

From a normal population whose mean is, say, μ and standard deviation is, say, σ, a number of samples may be collected, from which respective sample means, say $(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_i, \ldots)$ can be computed. The difference between each sample mean and population mean can be thought of as an error. However, in practice and due to various reasons, an experimenter selects randomly only one sample of size, say, n, and computes a sample mean, say $\bar{X}$. Then the difference $|\bar{X} - \mu|$ is treated as an absolute error. Apart from this, a $(1 - \alpha)100\%$ confidence interval for μ can be constructed by setting

$$\mathrm{P}\left(-z_{\alpha/2} \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le +z_{\alpha/2}\right) = (1-\alpha)100\%$$

Here α is the level of significance or the probability of Type-I error. Therefore, an experimenter always prefers to fix the sample size n such that the absolute error is kept at minimum, that is, $|\bar{X} - \mu| \le \varepsilon$, $\varepsilon > 0$ with maximum confidence that can result from maintaining minimum Type-I error probability. Clearly, $\varepsilon = z_{\alpha/2}\,\sigma/\sqrt{n}$ and hence $n = \left(z_{\alpha/2}\,\sigma/\varepsilon\right)^2$.

Since the population standard deviation σ is usually unknown and the sample standard deviation cannot be used as it needs the sample size n, there is a difficulty in determining the sample size n. In this paper, under the normality assumption, it is proposed to estimate the unknown population standard deviation from the specification of the quality characteristic that is under study from the perspective of Six Sigma quality (SSQ), which can ensure only 3.4 defects per million opportunities (DPMO); refer to Ravichandran (2006). This estimated standard deviation is then used to determine the sample size.

Process and product specifications play a major role in ensuring the degree of quality of a process or product. It may be noted that a unit of a product is said to be defective if it fails to meet the preset specification limits of the quality characteristic that is critical-to-quality. Setijono (2010) has considered the case of matching the SSQ limits to specification limits in order to estimate customer dissatisfaction (not meeting specification) and delight (meeting specification) in a survey related study. A similar study was done by Ravichandran (2016) from the perspective of process/product specification to estimate DPMO and extremely good parts per million opportunities (EGPMO) for higher the better and lower the better quality characteristics. A process or product that meets the specification target is always said to be stable. However, the process/product mean may move away from the target over a period of time. In the context of Six Sigma, this has prompted the practitioners to allow a shift up to $\pm 1.5\sigma$ (Lucas, 2002) as it can still produce only 3.4 DPMO. It has been argued that, though such a shift from the target is not acceptable to many researchers due to lack of either theoretical or empirical justification (Antony, 2004), there is a strong belief among the Six Sigma practitioners that no process can maintain on its own target in the long run. Therefore, the population mean and standard deviation estimated using the proposed method are expected to satisfy the Six Sigma goal of 3.4 DPMO.

Sampling from Normal Population

Let the quality characteristic X follow a normal distribution with mean μ and variance σ^2. That is,

$$\mathrm{E}(X) = \mu \quad \text{and} \quad \mathrm{V}(X) = \sigma^2$$

Let $(x_1, x_2,\ldots, x_i,\ldots, x_n)$ be a sample of size n drawn from this population. Then the sample mean $\bar{X}$ and sample variance S^2 are given as

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{1}$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{X}\right)^2 \tag{2}$$

It may be noted that the sample mean and variance given in (1) and (2) are the unbiased estimators of the mean μ and variance σ^2, respectively. That is,

$$\mathrm{E}\left(\bar{X}\right) = \mu \quad \text{and} \quad \mathrm{E}\left(S^2\right) = \sigma^2$$

Because the sample mean $\bar{X}$ itself can be thought of as a random variable as it can vary for varying samples, the mean and variance of the sample mean itself can be shown as μ and σ^2/n. It is a proven result that the sample mean $\bar{X}$ also follows the normal distribution with mean μ but with variance σ^2/n. In general, the standard deviation σ/n of the sample mean $\bar{X}$ is known as standard error (SE).

Standard Normal Distribution

It may be recalled that if the underlying distribution of the random variable X has mean μ and known variance σ^2, then we can define a standard normal variate, say Z, as

$$Z = \frac{X-\mu}{\sigma} \quad \text{or} \quad Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \tag{3}$$

or in general, equation (3) can be written as

$$Z = \frac{\text{sample statistic} - \mathrm{E}\left(\text{sample statistic}\right)}{\mathrm{SE}\left(\text{sample statistic}\right)}$$

which has mean 0 and variance 1. Here, E(*) represents expectation and SE(*) represents standard error. However, if the standard deviation σ is unknown then Z is observed to be not a standard normal variate. Under this circumstance, we replace the unknown standard deviation σ by the sample standard deviation given by S and construct a variable called Student's T as

$$T = \frac{\bar{X}-\mu}{S/\sqrt{n}} \tag{4}$$

which follows the Student's T distribution with $n - 1$ degrees of freedom. It may also be noted that the Student's T variable is defined when the sample size n is small.

Error and Sample Size

It may be noted that fixing the sample size n is a major concern in statistical inference problems. As discussed earlier, a large sample size, though preferred, may be expensive, laborious, and time-consuming, while a small sample may result in poor and inconsistent inferential decisions. Statistical errors – Type-I and Type-II errors – are also influenced by the size of the sample. Therefore, there needs to be a balance between these two types of error. It is preferable to choose n such that the *size*, say α, which is the probability of Type-I error and *power*, say $1 - \beta$, where β is the probability of Type-II error, are optimum and vice-versa. Given the Type-I error probability α, it is known that

$$\begin{aligned} &\mathrm{P}\left(-z_{\alpha/2} \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le +z_{\alpha/2}\right) = (1-\alpha)100\% \\ &\Rightarrow \mathrm{P}\left(\left|\bar{X}-\mu\right| \le +\frac{\sigma}{\sqrt{n}} z_{\alpha/2}\right) = (1-\alpha)100\% \end{aligned} \tag{5}$$

Here $\pm z_{\alpha/2}$ can be obtained by setting $P(Z < -z_{\alpha/2}) = P(Z > +z_{\alpha/2}) = \alpha/2$ with an assumed value of $\mu = \mu_0$ (null hypothesis is true), and hence $Z \sim N(0, 1)$. Now it is supposed that an experimenter would like to have the difference (*error*) between the sample mean $\bar{X}$ and the unknown population mean μ to be less than or equal to a pre-specified negligible value, say ε (> 0), with the confidence level $(1 - \alpha)100\%$. This implies that

$$\frac{\sigma}{\sqrt{n}} z_{\alpha/2} = \varepsilon \quad \Rightarrow \quad n = \left(\frac{\sigma}{\varepsilon} z_{\alpha/2}\right)^2 \tag{6}$$

(Refer to Montgomery & Runger, 2003; Ravichandran, 2010). One way of choosing ε is to allow the difference between $\bar{X}$ and μ as some δ (> 0) percentage of μ, that is $\varepsilon = (\delta/100)\mu$. Therefore we have

$$n = \left(\frac{\sigma}{(\delta/100)\mu} z_{\alpha/2} \right)^2 = \left\{ \frac{1}{\delta} \left(\frac{\sigma}{\mu} 100 \right) z_{\alpha/2} \right\}^2 \tag{7}$$

Accordingly, if μ and σ are known, then for the known values of $(\sigma/\mu)100$ (note that $(\sigma/\mu)100$ gives the coefficient of variation (CV)) and for different δ values, the sample size can be determined by fixing α values. Table 1 shows such sample size values for

(i) CV = $(\sigma/\mu)100$ = 2.5% (2.5) 20%
(ii) δ = 1.0, 2.5, 50
(iii) α = 0.01, 0.05, 0.10

Readers may note that, in Table 1, CV = x% and $\delta = y$ means $x = \sigma/\mu$ and $y = \delta/100$ so that

$$n = \left\{ \frac{x}{y} z_{\alpha/2} \right\}^2$$

Table 1 can now readily be used by the experimenters for sampling or can be used as a guideline for determining sample size for other combinations of parameters. If both Type-I and Type-II error probabilities are known, then the sample size n given in equation (6) can also be written as

$$n \approx \left(\frac{\sigma}{\varepsilon} \left[z_{\alpha/2} + z_\beta \right] \right)^2 \tag{8}$$

Here z_β can be obtained by setting β equal to $\mathrm{P}\left(Z < +z_{\alpha/2} - \varepsilon\sqrt{n}/\sigma\right) - \mathrm{P}\left(Z < -z_{\alpha/2} - \varepsilon\sqrt{n}/\sigma\right)$ with an assumed mean value $\mu = \mu_1 = \varepsilon\sqrt{n}/\sigma$ (alternative hypothesis is true) and hence $Z \sim \mathrm{N}\left(\varepsilon\sqrt{n}/\sigma, 1\right)$. It is observed that the approximation in (8) holds good if $\mathrm{P}\left(Z \le -z_{\alpha/2} - \varepsilon\sqrt{n}/\sigma\right)$ is small (= 0) compared to β for the sample size given in (8). Refer to Montgomery and Runger (2003) for more details. Therefore, $\mathrm{P}(Z < -z_\beta) = \beta$ implies that $-z_\beta = +z_{\alpha/2} - \varepsilon\sqrt{n}/\sigma$. Following (7), we have

Table 1. Sample size values according to equation (7)

CV	δ	Size (α) 0.01	0.05	0.10
2.5	1.0	42	24	17
	2.5	7	4	3
	5.0	2	1	1
5.0	1.0	166	96	68
	2.5	27	15	11
	5.0	7	4	3
7.5	1.0	374	216	153
	2.5	60	35	25
	5.0	15	9	6
10.0	1.0	666	384	272
	2.5	107	61	44
	5.0	27	15	11
12.5	1.0	1040	600	425
	2.5	166	96	42
	5.0	42	24	17
15.0	1.0	1498	864	613
	2.5	240	138	98
	5.0	60	35	25
17.5	1.0	2039	1176	834
	2.5	326	133	82
	5.0	82	47	33
20.0	1.0	2663	1537	1089
	2.5	426	246	174
	5.0	107	61	44

$$n \approx \left\{ \frac{1}{\delta} \left(\frac{\sigma}{\mu} 100 \right) \left[z_{\alpha/2} + z_{\beta} \right] \right\}^2 \tag{9}$$

From Table 1, the following observations can easily be made:

(i) For a fixed CV, as the error δ increases, the sample size n decreases meaning that smaller sample size will result in higher error and vice-versa.

(ii) For a fixed CV, as the Type-I error probability α increases, the sample size n decreases meaning that smaller sample size will result in higher degree of Type-I error probability (size) and vice-versa.

(iii) As CV increases, the sample size increases and vice-versa. This means that if the CV is less, then fewer sample observations are sufficient to achieve the error levels.

If μ is zero, then it is always wise to use the formula involving ε given in equation (6) rather than using the formula involving $\delta\mu$ given in (7). Values for ε can be assumed to be 10^{-2}, 10^{-3}, 10^{-4}, etc. If σ is unknown, S cannot be used in (6) or (7) since $E(S) \neq \sigma$. But, though $E(S/c_4) = \sigma$ where c_4 is an appropriate constant, one cannot use c_4 and S since both of them depend on sample size n. Therefore, using

$$n = \left(\frac{S/c_4}{\varepsilon} t_{v,\alpha/2} \right)^2 \quad \text{or} \quad n = \left(\frac{S/c_4}{(\delta/100)\mu} t_{v,\alpha/2} \right)^2 \tag{10}$$

respectively, as replacement of (6) or (7) for sample size determination is erroneous.

Stopping Criteria in Simulations

There are situations, such as simulations, where it is important to decide when to stop the simulation. Under these circumstances, the simulations are run for a preset number n_1 of times (i.e., sample of size n_1) and then the sample mean $\bar{X}_1$, standard deviation S_1, c_4^1, and $t_{v_1,\alpha/2}$ are computed for the quality characteristic of interest, say X. The simulation is stopped if the following condition is satisfied (refer to Yeap, 1998):

$$n_1 \geq \left(\frac{S_1/c_4^1}{\varepsilon} t_{v_1,\alpha/2} \right)^2 \quad \text{or} \quad n_1 \geq \left(\frac{S_1/c_4^1}{(\delta/100)\bar{X}_1} t_{v_1,\alpha/2} \right)^2$$

Otherwise, collect the next observation from the next simulation so that $n_2 = n_1 + 1$, from which $\bar{X}_2$, S_2, c_4^2, and $t_{v_2,\alpha/2}$ are computed to verify if

$$n_2 \geq \left(\frac{S_2/c_4^2}{\varepsilon} t_{v_2,\alpha/2} \right)^2 \quad \text{or} \quad n_2 \geq \left(\frac{S_2/c_4^2}{(\delta/100)\bar{X}_2} t_{v_2,\alpha/2} \right)^2$$

In general, the simulation is stopped after n_i, $i = 1, 2, \ldots$, simulations if

$$n_i \geq \left(\frac{S_i/c_4^i}{\varepsilon} t_{v_i,\alpha/2} \right)^2 \quad \text{or} \quad n_i \geq \left(\frac{S_i/c_4^i}{(\delta/100)\bar{X}_i} t_{v_i,\alpha/2} \right)^2, \quad i = 1, 2, \ldots$$

where the mean $\bar{X}_i$, standard deviation S_i, c_4^i, and $t_{v_i,\alpha/2}$ are computed form the sample of size n_i, that is after i simulations.

A method is proposed here to estimate the unknown population standard deviation σ from the perspective of the concept of SSQ. This sample size can ensure the conformance of the process to the Six Sigma goal of 3.4 DPMO.

Sample Size Determination based on Six Sigma Quality

Consider a measurable quality characteristic, say X, that follows normal process with mean $T = \mu$ and variance σ^2. Because not all values of X towards the tails of the distribution are acceptable, the specification of X is usually given in the form $T \pm K\sigma$, where T is the target or population mean, K is a positive constant, and σ is the population standard deviation. Notationally, $X \sim N(T, \sigma^2)$ and $P(T - K\sigma \leq X \leq T + K\sigma) = 1 - \alpha_K$, where α_K is a prespecified probability value such that $\alpha_K = P(X < T - K\sigma) + P(X > T + K\sigma)$. From $T \pm K\sigma$, we get half of the process spread as $K\sigma = d$ (say) (also refer to Lin, 2006), which implies $\sigma = d/K$ and hence we have $\hat{\sigma}_{SS} = \sigma = d/K$. Therefore, we have $\hat{\sigma}_{SS}/\sqrt{n_{SS}} = (d/K)/\sqrt{n_{SS}}$. Now equation (5) becomes

$$\Rightarrow P\left(\left| \bar{X} - \mu \right| \leq + \frac{d/K}{\sqrt{n_{SS}}} z_{\alpha_K/2} \right) = (1 - \alpha_K)100\% \tag{11}$$

and hence equations (6) and (7) become, respectively:

$$n_{SS} = \left(\frac{d/K}{\varepsilon} z_{\alpha_K/2} \right)^2 \tag{12}$$

Table 2. Determination of α_K and $z_{\alpha_K/2}$

K	DPMO	α_K	$z_{\alpha_K/2}$
3.0	66810.63	0.1336210	1.50
3.5	22750.35	0.0455010	2.00
4.0	6209.70	0.1241900	2.50
4.5	1349.97	0.0027000	3.00
5.0	232.67	0.0004650	3.50
5.5	31.69	0.0000634	4.00
6.0	3.40	0.0000068	4.50

and

$$n_{\mathrm{SS}} = \left(\frac{d/K}{(\delta/100)\,\mu} z_{\alpha_K/2} \right)^2 \tag{13}$$

Here, K represents the current sigma quality level (SQL) of the process. For example, if $K = 6$, then we have DPMO = 3.4 either on left tail or on right tail. Therefore, $\alpha_K = 6.8 \times 10^{-6}$ implies $z_{\alpha_K/2} = 4.50$. In (13), if μ is unknown, then the same can be replaced by the specification target T.

The computation of the values of $z_{\alpha_K/2}$ with different SQLs is discussed as follows: If the process is operating at a Three Sigma level, then we have the current quality level as $K = 3$. It may be noted that, with allowable shift, a Three Sigma process may result in 66810.63 DPMO. Once this level is maintained, and if there is a scope for improvement, the practitioner may change the value of $z_{\alpha_K/2}$. Various DPMOs and the corresponding $z_{\alpha_K/2}$ values are given as shown in Table 2 (Harry, 1998; Lucas, 2002). Therefore, for SSQ process with 3.4 DPMO, (12) and (13) respectively become

$$n_{\mathrm{SS}} = \left(\frac{d/6}{\varepsilon} (4.50) \right)^2 \tag{14}$$

and

$$n_{SS} = \left(\frac{d/6}{(\delta/100)\,\mu}(4.50) \right)^2 \quad \text{or} \quad n_{SS} = \left(\frac{1}{\delta}\left(\frac{d/6}{\mu}100 \right)4.50 \right)^2$$
$$\Rightarrow n_{SS} = \left(\frac{1}{\delta}\left(\frac{\hat{\sigma}_{SS}}{\mu}100 \right)4.50 \right)^2 \tag{15}$$

Shown in Table 3 are sample size values from the perspective of Three Sigma (3σ), Four Sigma (4σ), Five Sigma (5σ), and Six Sigma (6σ) qualities for the following parameter set up:

(i) $CV_{SS} = (\hat{\sigma}_{SS}/\mu)x100 = 1.0, 2.5\%(2.5)20\%$

(ii) δ = 1.0, 2.5, 5.0

(iii) α_K = 0.1336210, 0.1241900, 0.0004650, 0.0000068

From Table 3, it can be seen that:

(i) For a fixed CV_{SS}, as the error δ increases, the sample size n decreases meaning that a smaller sample size will result in higher error and vice-versa.

(ii) For a fixed CV_{SS}, as the sigma quality decreases (that is, as the Type-I error probability α increases), the sample size n decreases meaning that a smaller sample size will result in poor sigma quality and vice-versa.

(iii) As CV increases the sample size increases and vice-versa. This means that if the CV is less, then fewer sample observations are sufficient to achieve the goal of SSQ of 3.4 DPMO. For example, if $(d/6)/\mu = 0.01$ and the error percentage is $\delta = 1\%$ of μ, then an inspection of a sample with 20 observations is sufficient to show if the process is meeting the Six Sigma goal of 3.4 DPMO.

Table 3 is an indicative one, and experimenters can use it as a guideline for determining the sample size for different parameter combinations. Looking at Tables 1 and 3, the values of

$$CV = \frac{\sigma}{\mu}100\% \quad \text{and} \quad CV_{SS} = \frac{\hat{\sigma}_{SS}}{\mu}100\%$$

are assumed as same for comparison purpose. However, in practice, the variation indicated by $\hat{\sigma}_{SS}$ in the case of a Six Sigma process is usually far below the normal process whose variation is indicated by σ. Therefore, reduced variation in Six Sigma may result in a good reduction in the sample size. See example 2 in the following section.

Table 3. Sample size n_{SS} for Six Sigma quality

CV_{SS}	δ	6σ	5σ	4σ	3σ
1	1.0	20	12	6	2
	2.5	3	2	1	-
	5.0	1	-	-	-
2.5	1.0	127	77	39	14
	2.5	20	12	6	2
	5.0	5	3	2	1
5.0	1.0	506	306	156	56
	2.5	81	49	25	9
	5.0	20	12	6	2
7.5	1.0	1139	689	352	127
	2.5	182	110	56	20
	5.0	46	28	14	5
10.0	1.0	2025	1225	625	225
	2.5	324	196	100	36
	5.0	81	49	25	9
12.5	1.0	3164	1914	977	352
	2.5	506	306	156	56
	5.0	127	77	39	14
15.0	1.0	4556	2756	1406	506
	2.5	729	441	225	81
	5.0	182	110	56	20
17.5	1.0	6202	3752	1914	689
	2.5	992	600	306	110
	5.0	248	150	77	28
20.0	1.0	8100	4900	2500	900
	2.5	1296	784	400	144
	5.0	324	196	100	36

Numerical Examples

Example 1

Yeap (1998) has given an example that the standard deviation of power samples measured from a circuit has been observed to have $\pm$ 20% fluctuations from the mean. Now the number of sample units (sample size) required to ensure that the experimenter is 99% confidence that the error of the sample mean is within $\pm$ 5% can be obtained by setting:

$$\sigma = 20\%\mu \Rightarrow \sigma/\mu = 0.2, \quad \delta = 5\% = 5/100$$

which, according to (7), gives

$$n = \left\{ \frac{1}{\delta} \left(\frac{\sigma}{\mu} 100 \right) z_{\alpha/2} \right\}^2 = \left\{ \frac{1}{5} (0.2)(100)(2.58) \right\}^2 \approx 107$$

However, for the SSQ requirement of 3.4 DPMO, the sample size can be obtained as

$$n_{\mathrm{SS}} = \left\{ \frac{1}{\delta} \left(\frac{\hat{\sigma}_{\mathrm{SS}}}{\mu} 100 \right) z_{\alpha_K/2} \right\}^2 = \left\{ \frac{1}{5} (0.2)(100)(4.50) \right\}^2 \approx 324$$

It is alarming to note that the SSQ process requires more sample observations in this example. This is due to the fact that the CV% is too high with $\sigma = 20\%\mu$, which is beyond expectation. However, it is presented here for illustration purpose to show that given this CV% and the specification of the quality characteristic of interest, it may require 324 sample observations to ensure that it is a Six Sigma process.

Example 2

Montgomery and Runger (2003) presented an example of vane-manufacturing process. The specifications on vane opening are given as 0.5030 $\pm$ 0.0010 inches. Let us suppose that we would like to draw a sample of size n so that the process average can lie around $\pm$ 0.05% of the target. Then the sample size meeting the SSQ requirement of 3.4 DPMO can be obtained by setting:

$$\mu = 0.5030, \quad \hat{\sigma}_{\mathrm{SS}} = d/6 = 0.0010/6 = 0.000167,$$
$$\sigma/\mu = 0.000361, \quad \delta = 0.05\% = 0.05/100$$

This, according to (15), gives

$$n_{\mathrm{SS}} = \left\{ \frac{1}{\delta} \left(\frac{\hat{\sigma}_{\mathrm{SS}}}{\mu} 100 \right) z_{\alpha_K/2} \right\}^2 = \left\{ \frac{1}{0.05} (0.0361)(4.50) \right\}^2 \approx 11$$

If it is assumed that by past experience the standard deviation of this process is known as 0.00025, then the required sample size can be obtained by setting:

$$\mu = 0.5030, \sigma = 0.00025, \sigma/\mu = 0.000497, \delta = 0.05\% = 0.05/100$$

$$n = \left\{ \frac{1}{\delta} \left(\frac{\sigma}{\mu} 100 \right) z_{\alpha/2} \right\}^2 = \left\{ \frac{1}{0.05} (0.000497)(100)(4.50) \right\}^2 \approx 20$$

It may be noted that since σ = 0.00025, the process is at the level of 4σ only with K = 4 (that is, 4σ = (4)(0.00025) = 0.0010 = d) and hence it requires more sample observations. Therefore, the process variation needs to be improved (reduced variation) with regard to standard deviation from σ = 0.00025 to σ = 0.000167 so that the process becomes a Six Sigma process with 3.4 DPMO.

If an experimenter is interested in drawing a sample of size n so that it meets the Four Sigma requirement of 6209.70 DPMO, then it can be obtained by setting:

$$\mu = 0.5030, \quad \hat{\sigma}_{\mathrm{SS}} = d/4 = 0.0010/4 = 0.00025,$$
$$\hat{\sigma}_{\mathrm{SS}}/\mu = 0.000497, \quad \delta = 0.05\% = 0.05/100$$

$$n_{\mathrm{SS}} = \left\{ \frac{1}{\delta} \left(\frac{\hat{\sigma}_{\mathrm{SS}}}{\mu} 100 \right) z_{\alpha_K/2} \right\}^2 = \left\{ \frac{1}{0.05} (0.000497)(100)(2.50) \right\}^2 \approx 6$$

Given the process conditions, it may be noted that a meager sample of size 6 is sufficient to meet the error constraints under Four Sigma quality of 6209.70 DPMO.

Example 3

Consider an example of a manufacturing process of a product in which the specification for the dimension of the product is set as 20 ± 6. For laboratory testing purposes it is proposed to collect sample units of the product. The error limit between sample mean and the target is set as $\pm 5\%$ of the target. Then the sample size meeting the SSQ requirement of 3.4 DPMO can be obtained by setting:

$$\mu = 20, \hat{\sigma}_{\mathrm{SS}} = d/6 = 6/6 = 1, \hat{\sigma}_{\mathrm{SS}}/\mu = 0.005, \delta = 5\% = 5/100$$

This, according to (15), gives

$$n_{\mathrm{SS}} = \left\{ \frac{1}{\delta} \left(\frac{\hat{\sigma}_{\mathrm{SS}}}{\mu} 100 \right) z_{\alpha_K/2} \right\}^2 = \left\{ \frac{1}{5} (0.05)(100)(4.50) \right\}^2 \approx 20$$

This can also be verified from Table 3. Now, after drawing a sample of size 20, the sample standard deviation is computed as 3.63, which is an indication that the process is only at an SQL of 6/3.63 = 1.65 sigma. Therefore, the process variation needs to be improved (reduced variation) with regard to standard deviation from 3.63 to 1 so that the process becomes a Six Sigma process with 3.4 DPMO.

Discussions and Conclusions

In this paper, first a discussion on the existing methods of sample size determination is presented. It is observed that such methods critically need the known population standard deviation. Therefore, a new approach is then presented that uses an estimate of population standard deviation from the perspective of the Six Sigma goal of 3.4 DPMO. The proposed method helps the experimenter to fix the sample size in such a way that the process either meets the SSQ requirement of 3.4 DPMO or can be improved towards the goal. This can be achieved by comparing the estimated standard deviation from the perspective of Six Sigma and the actual process standard deviation obtained after fixing the sample size. If the difference is wide, then we recommend using the stopping criteria approach by adding more samples until the requirements are met.

The proposed sample size determination method is studied and evaluated numerically. It is observed that as the CV% increases, the method recommends a larger sample size to cover up the higher standard deviation and vice-versa. The approach is also demonstrated using suitable examples. In these examples, it is discussed that the proposed method not only helps in determining the sample size, it also prompts the experimenter to look for improvement opportunities, such as variance reduction exercises through quality improvement programs. As a future study, the case of proportions instead of measurable quality characteristic will be considered. Also, it will be attempted to propose a method for determining a sample of specific size from a finite size population.

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